

Imitating Chemical Motors with Optimal Information Motors

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To induce transport, detailed balance must be broken. A common mechanism is to bias the dynamics with a thermodynamic fuel, such as chemical energy. An intriguing, alternative strategy is for a Maxwell demon to effect the bias using feedback. We demonstrate that these two different mechanisms lead to distinct thermodynamics by contrasting a chemical motor and information motor with identical dynamics. To clarify this difference, we study both models within one unified framework, highlighting the role of the interaction between the demon and the motor. This analysis elucidates the manner in which information is incorporated into a physical system.

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Information is physical [1]: it must be recorded in a physical memory, and therefore the processing of information is constrained by the same thermodynamic limitations as any other physical process [2, 3]. Remarkably, once that information has been obtained, it can then serve as a thermodynamic resource similar in character to free energy, as first pointed out by Szilard [4]. These two observations have been the historic points of departure for investigations into the nature of information [2, 3]. As a consequence, research has either focused on the manipulation of information in isolated memories, or simply on the engines that utilize that information. This division has been fruitful. Theoretical studies of isolated memories, which have been verified by experiment [5], have led to insights into the thermodynamic costs of measurement [6] and erasure [7–12]; copying [13, 14]; and proof-reading [13]; whereas, theoretical [15–21] and experimental [22, 23] investigations of information (or feedback) motors have explored the fundamental limits to the conversion of information into work.

Nevertheless, the thermodynamic qualities of information still need to be clarified, especially the precise mechanism that allows a motor to exploit the information stored in a memory to rectify thermal fluctuations. With this goal in mind, we highlight in this Letter the difference between information and a more traditional thermodynamic resource, the chemical free energy. We elaborate this distinction by comparing the entropy production rates of two motors with *identical* dynamics: a chemical motor powered by a chemical potential energy gradient and an information motor driven by feedback. Despite the identical dynamics, the information motor presents qualitatively different thermodynamics. We trace this discrepancy to the features of the interaction between the motor and the memory. We find that in order for the information to be a useful thermodynamic resource, this

interaction must create long-lived correlations.

Our motors are patterned on the Brownian ratchet [24] pictured in Fig. 1. The ratchet is composed of a particle

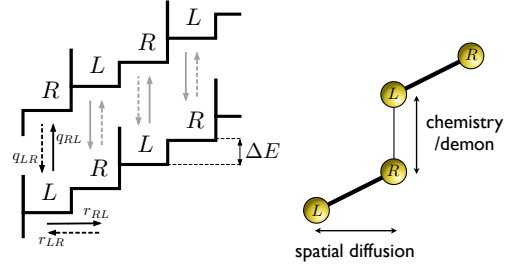


FIG. 1. Depiction of the Brownian ratchet template. A particle moves in a periodic potential of period l against a force $F = 2\Delta E/l$ with rates r_{LR} and r_{RL} . The potential makes random switches between two configurations mediated by either a chemical reaction (chemical motor) or a demon (information motor). Potential switches induce transitions $R \leftrightarrow L$ at rates q_{LR} and q_{RL} .

driven against a force F by a flashing periodic potential. The potential fluctuates between two configurations consisting of a series of offset infinite barriers that confine the particle to boxes of length l . Within each box the particle has two spatial states: $z = L$ (left) and a higher energy state R (right) with energy difference $\Delta E = Fl/2$. We model the transitions between R and L as a discrete Markov jump process [25], and we set the units of energy by fixing the temperature, $kT = 1$. Diffusive jumps *within* each box are thermally activated with rates r_{LR} (r_{RL}) from R to L (L to R) that verify detailed balance, $r_{LR}/r_{RL} = e^{\Delta E}$ [26]. Transitions effected by switching the potential's configuration occur with rates q_{LR} (q_{RL}) for $R \rightarrow L$ ($L \rightarrow R$) and are induced by a *different mechanism in each motor*, see Fig. 1. Furthermore, these tran-

sitions do not require energy, as we assume that R (L) in the lower configuration in Fig. 1 has the same energy as L (R) in the upper. The ratchet functions as long as the $R \rightarrow L$ transitions occur more often than the reverse, $L \rightarrow R$. In which case, the current J – the average net number of jumps per unit time against the load – is positive, and work is extracted at a rate

$$\dot{W}_{\text{ext}} = J\Delta E. \quad (1)$$

It will prove convenient in our subsequent calculations to assume that potential switches are slow compared to the spatial transitions ($q \ll r$). In this limit, the particle has sufficient time between successive switches of the potential to relax to the equilibrium probability density

$$p_R = \frac{1}{1 + e^{\Delta E}}, \quad p_L = \frac{e^{\Delta E}}{1 + e^{\Delta E}}. \quad (2)$$

Consequently, $J = q_{LR}p_R - q_{RL}p_L$.

In the chemical motor, the potential switches are biased by coupling them to an out-of-equilibrium chemical reaction between species A and B through the formula $R + A \leftrightarrow L + B$. Detailed balance enforces that the chemical potential energy difference between A and B , $\Delta\mu \equiv \mu_A - \mu_B > 0$, satisfies $\Delta\mu = \ln(q_{LR}/q_{RL})$. The resulting kinetic-diffusion scheme corresponds to the minimal tight-coupled chemical motor extensively used to model protein motors and pumps [26]. When $\Delta\mu > \Delta E$, \dot{W}_{ext} [Eq. (1)] is extracted by consuming chemical free energy per unit time $\dot{F}_{\text{chem}} = J\Delta\mu$. The resulting entropy production rate is [26]

$$\dot{S}^{(\text{chem.mot.})} = \dot{F}_{\text{chem}} - \dot{W}_{\text{ext}} = J(\Delta\mu - \Delta E) \geq 0. \quad (3)$$

To contrast with the chemical motor, we now consider an information motor driven by feedback implemented by a device, or so-called demon, that switches the potential in response to measurements of the particle's position. Here, the information is the resource that powers the ratchet, whereas chemical energy was used by the chemical motor.

In order for the information motor to reproduce the stochastic potential switches of the ratchet, the demon must perform measurements at random times separated by intervals sampled from a Poisson distribution with rate $\alpha = q_{RL} + q_{LR}$ [25]. This scheme may be interpreted as the demon attempting to make a measurement in each small interval of time δt , but only succeeding with probability $\alpha\delta t \ll 1$. When the demon succeeds, it measures R or L with a symmetric error, mistaking R (L) for L (R) with probability $\epsilon = q_{RL}/(q_{RL} + q_{LR})$, and flips the potential when the outcome is R . Moreover, the demon records the sequence of potential switches in a memory with states m , though for now we leave unspecified the recording mechanism. When the demon fails to make a measurement or the outcome is L , the memory is put in

state $m = N$ for no-switch; whereas, when the outcome is R , the memory is set to S for switch.

With this setup, potential switches occur at rates $q_{LR} = \alpha(1 - \epsilon)$ and $q_{RL} = \alpha\epsilon$, as desired. Comparison with the chemical motor leads to the correspondence $\Delta\mu = \ln[(1 - \epsilon)/\epsilon]$: $\epsilon = 0$ is equivalent to $\Delta\mu = \infty$, whereas $\epsilon = 1/2$ corresponds to an equilibrium fuel, $\Delta\mu = 0$.

We now need to determine the information motor's entropy production rate. For this we will use the framework introduced in Refs. [12, 20, 27–33] for the analysis of measurement and feedback. We will validate this approach later using a concrete physical realization. The framework's main features can be simply obtained by introducing a nonequilibrium free energy [34, 35] for a system whose mesoscopic states x are in local equilibrium [36]. To each system configuration $X = \{p_x, F_x\}$ characterized by the free energy F_x and probability p_x to be in x , we assign a *nonequilibrium free energy*

$$\mathcal{F}(X) = \sum_x p_x F_x - kTH(X) \equiv F(X) - kTH(X), \quad (4)$$

where $H(X)$ is the Shannon entropy [37]. We call $F(X) = \sum_x p_x F_x$ the *bare free energy*. In equilibrium, $p_x^{\text{eq}} = e^{-\beta F_x}/Z$ with $Z = \sum_x e^{-\beta F_x}$, and we recover the equilibrium free energy $\mathcal{F}^{\text{eq}} = -kT \ln Z$. The utility of \mathcal{F} stems from the observation that the (irreversible) entropy production in a transition between nonequilibrium configurations is the amount by which the work W exceeds the increment in the nonequilibrium free energy $\Delta\mathcal{F}$ ($kT = 1$) [34, 35]:

$$\Delta_i S = W - \Delta\mathcal{F} \geq 0. \quad (5)$$

Here, we are interested in the entropy production for the coupled memory and ratchet, $X = (M, Z)$, during measurement and feedback. An ideal classical measurement correlates the initially uncorrelated memory and ratchet, $\mathcal{F}(M, Z) = \mathcal{F}(M) + \mathcal{F}(Z)$, through an isothermal process *without* affecting the ratchet, $\mathcal{F}(Z') = \mathcal{F}(Z)$, though possibly changing the nonequilibrium free energy of the memory to $\mathcal{F}(M') \neq \mathcal{F}(M)$ [6, 12]. Upon completion of the measurement the nonequilibrium free energy in Eq. (4) can be cast into a useful form using the mutual information [37]

$$I(M', Z) = H(M') + H(Z) - H(M', Z), \quad (6)$$

as

$$\mathcal{F}(M', Z) = \mathcal{F}(M') + \mathcal{F}(Z) + I(M', Z), \quad (7)$$

assuming that before and after the measurement the bare free energies are additive, $F(M, Z) = F(M) + F(Z)$. The mutual information is a measure of correlations satisfying $I \geq 0$ with $I = 0$ only when M' and Z are independent [37]. Consequently, the creation of correlations, or

measuring, increases the nonequilibrium free energy, requiring work W_{meas} and producing entropy in accordance with Eqs. (5) and (7) [6, 12, 33],

$$\begin{aligned}\Delta_i S_{\text{meas}} &= W_{\text{meas}} - \Delta \mathcal{F}(M, Z) \\ &= W_{\text{meas}} - \Delta \mathcal{F}(M) - I(M', Z) \geq 0,\end{aligned}\quad (8)$$

where $\Delta \mathcal{F}(X) = \mathcal{F}(X') - \mathcal{F}(X)$.

Once the correlations have been established, they can be exploited through an isothermal process that extracts work W_{ext} from the ratchet by changing its nonequilibrium free energy to $\mathcal{F}(Z'') \neq \mathcal{F}(Z)$, without altering the memory, $\mathcal{F}(M'') = \mathcal{F}(M')$. This scenario, which we call feedback, ideally will produce an entropy [Eq. (5)] [27, 31, 33, 34]

$$\Delta_i S_{\text{fb}} = I(M', Z) - \Delta \mathcal{F}(Z) - W_{\text{ext}} \geq 0, \quad (9)$$

when all the correlations are removed. For cyclic feedback protocols, as we consider here, $\Delta \mathcal{F}(Z) = 0$. Only when the measurements are reversible ($\Delta_i S_{\text{meas}} = 0$) does Eq. (9) represent the total entropy production for the entire measurement and feedback cycle. In general, $\Delta_i S_{\text{fb}}$ is only a lower bound.

Now, since the information motor utilizes feedback, we can use Eq. (9) to calculate the (minimum) entropy production rate. From Eq. (9), we see that we must calculate I . To this end, we first observe that the fast diffusion implies that the ratchet begins each δt with the same equilibrium probability density [Eq. (2)]. Consequently, each interval is independent, and each N or S is recorded instantaneously in an independent memory. In this case, there are no correlations between successive measurement outcomes, allowing us to analyze each measurement separately. We then obtain I from the probability density $p'_{z,m}$ for the composite system at the end of an interval,

$$\begin{pmatrix} p'_{R,N} \\ p'_{R,S} \\ p'_{L,N} \\ p'_{L,S} \end{pmatrix} = \begin{pmatrix} p_R[1 - \alpha(1 - \epsilon)\delta t] \\ p_L\alpha\epsilon\delta t \\ p_L(1 - \alpha\epsilon\delta t) \\ p_R\alpha(1 - \epsilon)\delta t \end{pmatrix} = \begin{pmatrix} p_R(1 - q_{LR}\delta t) \\ p_Lq_{RL}\delta t \\ p_L(1 - q_{RL}\delta t) \\ p_Rq_{LR}\delta t \end{pmatrix} \quad (10)$$

Upon substituting this into Eq. (6) and expanding to first order in δt , we arrive at

$$\dot{I} = \frac{I(M', Z)}{\delta t} \simeq p_R q_{LR} \ln \frac{q_{LR}}{q_S} + p_L q_{RL} \ln \frac{q_{RL}}{q_S}, \quad (11)$$

where $q_S = (p'_{R,S} + p'_{L,S})/\delta t$ is the switching rate. Then by combining Eqs. (1), (9), and (11) with $\Delta \mathcal{F}(Z) = 0$, we find for the entropy production rate $\dot{S}^{(\text{info.mot.})} = \Delta_i S_{\text{fb}}/\delta t$ the form

$$\dot{S}^{(\text{info.mot.})} = \dot{I} - J\Delta E \geq 0. \quad (12)$$

In Fig. 2, we compare $\dot{S}^{(\text{chem.mot.})}$ [Eq. (3)] and $\dot{S}^{(\text{info.mot.})}$ [Eq. (12)] as functions of ΔE . Clearly, the

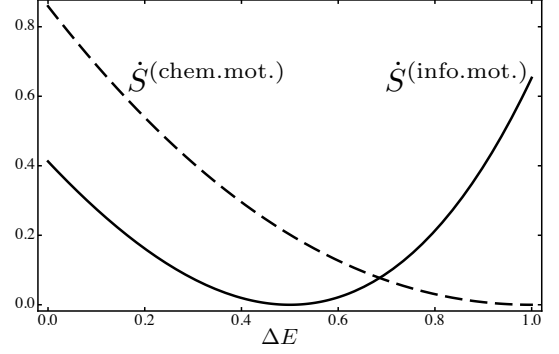


FIG. 2. Plot of the entropy production rates for the chemical motor, $\dot{S}^{(\text{chem.mot.})}$ [Eq. (3)], and the information motor, $\dot{S}^{(\text{info.mot.})}$ [Eq. (12)], as functions of the external force ΔE for $q_{RL} = 1$ and $\Delta\mu = \ln[(1 - \epsilon)/\epsilon] = 1$.

different switching mechanisms lead to qualitatively different thermodynamics, even though the dynamics are the same. Most notable is that the information motor can operate with zero entropy production at a *finite* current when $\Delta E = \Delta\mu/2$. By contrast, the chemical motor reaches the reversible limit only at the stall force $\Delta E = \Delta\mu$ when $J = 0$.

This analysis, however, ignores the interaction between the memory and ratchet, which obscures the dynamic transfer of information encoded in Eqs. (8) and (9). To clarify the role of this interaction, we now analyze an extension of the information motor where the memory and measurement recording mechanism are included explicitly, building on the mechanical Maxwell's demon introduced by Mandal and Jarzynski [38].

We model the memory as a tape composed of a series of two-state cells (or bits) with states $m = N, S$ with corresponding free energies $F_N = 0$ and $F_S = f_0 \rightarrow \infty$. Initially, each cell is in N – which is equilibrium ($p_N = 1$, $p_S = 0$). One at a time, each cell couples to the motor for a duration τ_1 , short compared to the diffusion ($r\tau_1 \ll 1$), through a very fast reaction that induces potential switches according to the scheme in Fig. 3. For the motor to function, we bias the $R \rightarrow L$ potential switches by mediating this transition with the same pair of out-of-equilibrium chemicals, A and B , used in the chemical motor with $\Delta\mu = \ln(q_{LR}/q_{RL})$ through the formula $R + N + A \leftrightarrow L + S + B$. Furthermore, during τ_1 we lower the free energy of S from $F_S = f_0$ to $f = -\ln(q_{RL}\delta t) \gg 1$ *quasistatically*, though fast compared to diffusion. Thus, the diffusion is frozen, while the reactions $R + S \leftrightarrow L + N$ and $R + N + A \leftrightarrow L + S + B$ independently evolve through a sequence of equilibrium states. Next, the cell decouples, and the motor relaxes to equilibrium by spatial diffusion for a time $\tau_2 \gg 1/r$ such that $\delta t = \tau_1 + \tau_2$ is the total cycle time. Repeating this sequence of actions – coupling a cell,

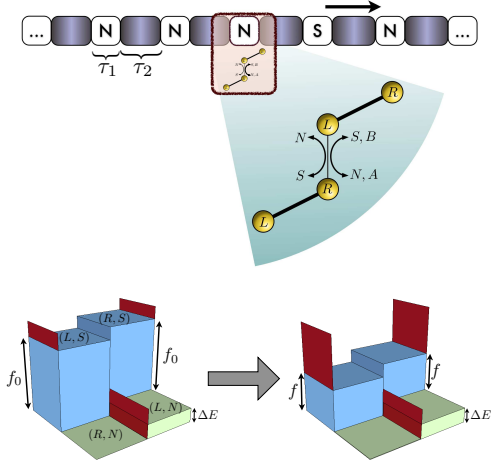


FIG. 3. Illustration of the extended information motor that is fed by a tape of two-state (N and S) cells each initially in N and separated by a time interval τ_2 (upper figure). Each cell couples to the ratchet for a duration τ_1 during which the free energy of S is quasistatically lowered from $f_0 \rightarrow \infty$ to f while the reactions $R + S \leftrightarrow L + N$ and $R + N + A \leftrightarrow L + S + B$ individually evolve in a time-dependent free energy landscape (lower figure).

lowering F_S , decoupling, and then relaxing diffusively – reproduces the dynamics of the ratchet, as can be verified by demonstrating that the joint density $p'_{z,m}$ at the end of the interval equals Eq. (10) to order δt .

The thermodynamic analysis of each δt -cycle naturally decomposes into two steps: the establishment of correlations, or measurement, during τ_1 , and the spatial relaxation during τ_2 when the correlations are converted into work.

During τ_1 , work is done by the $A \leftrightarrow B$ reaction, $W_{\text{chem}} = p'_{L,S} \Delta \mu$ [Eq. (10)], and by quasistatically lowering F_S from $f_0 \rightarrow \infty$ to f , which to order δt is

$$W_{\text{lower}} = \int_{f_0}^f \left[\frac{p_R}{1 + e^{f' - \Delta \mu}} + \frac{p_L}{1 + e^{f'}} \right] df' \simeq -p'_S, \quad (13)$$

where $p'_S = q_S \delta t = p'_{R,S} + p'_{L,S}$. This work is used to form correlations $I(M', Z)$ [Eq. (11)] while changing the memory's nonequilibrium free energy from $\mathcal{F}(M) = 0$ ($p_N = 0$) by $\Delta \mathcal{F}(M) = \mathcal{F}(M') = p'_S f - h(p'_S)$, where h is the binary Shannon entropy [37]. Inserting these expressions into Eq. (8), reveals, after a cumbersome though straightforward algebraic manipulation, that

$$\Delta_i S_{\text{meas}} = W_{\text{lower}} + W_{\text{chem}} - \Delta \mathcal{F}(M) - I(M', Z) = 0, \quad (14)$$

saturating the bound in Eq. (8). Our protocol is reversible because the initially equilibrium memory ($f_0 \rightarrow \infty$, $p_S = 0$) couples to an equilibrium ratchet, followed by a quasistatic isothermal shift in F_S .

The cycle is completed as the motor relaxes. I is converted into work $W_{\text{ext}} = J \delta t \Delta E$ through a decrease of

the nonequilibrium free energy to $\mathcal{F}(M'', Z'') = \mathcal{F}(M') + \mathcal{F}(Z)$. The resulting entropy production from Eq. (5) is $\Delta_i S_{\text{diff}} = I(M', Z) - J \delta t \Delta E \geq 0$, reproducing per cycle the entropy production rate of the information motor $\Delta_i S_{\text{diff}} / \delta t = \dot{S}^{(\text{info.mot.})}$. Thus, the total entropy production per cycle is $\Delta_i S_{\text{tot}} = \Delta_i S_{\text{meas}} + \Delta_i S_{\text{diff}} = \dot{S}^{(\text{info.mot.})} \delta t$, proving that we have a nonautonomous model for the information motor without explicit feedback that has the same dynamics *and* thermodynamics.

An insightful alternative formulation of $\Delta_i S_{\text{diff}}$ is through the conditional entropy production $\Delta_i S_{\text{diff}|m}$ for the subensemble with $m = N, S$: $\Delta_i S_{\text{diff}} = p'_S \Delta_i S_{\text{diff}|S} + (1 - p'_S) \Delta_i S_{\text{diff}|N}$. Since switches are rare ($p'_S \sim \delta t \ll 1$), the subensemble with $m = N$ deviates little from equilibrium, and therefore $\Delta_i S_{\text{diff}|N} \sim \delta t^2$ is negligible. On the other hand, $\Delta_i S_{\text{diff}|S} = \Delta E (p'_{R|S} - p_R) + h(p_R) - h(p'_{R|S})$, with $p'_{R|S} = p'_{R,S} / p'_S$, which vanishes only if $p'_{R|S} = p'_R$, or equivalently $\Delta E = \Delta \mu / 2$. In this situation, the probability density of the particle after measurement given a switch $p'_{R|S}$ is identical to equilibrium p_R . In which case, not only is the measurement reversible, but the entire operation of the composite system is reversible, in the same spirit as other reversible controlled systems analyzed in Refs. [20, 21].

In conclusion, we have demonstrated an explicit physical mechanism that allows information to be stored in a memory and later utilized. Central to this mechanism is the two-step interaction mediated by the tape that creates long-lived correlations [Eq. (10)] and leads to the appearance of the mutual information in the thermodynamic accounting [Eq. (14)]. The sequential structure of the tape, however, seems less important. We could also reproduce the same behavior using a chemical reservoir of molecules N and S , as long as there was a mechanism establishing stable correlations for fixed intervals. Remarkably, similar long-lived complexes are common in biology: they can be observed in the non-Poisson statistics of molecular motors [39], enzymatic catalysis [40], and sensory adaption [41]. It would be interesting to check if such complexes serve as a free energy storage and to uncover their role in information processing.

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